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**SOME METHODS FOR EXTRACTING
AERODYNAMIC DERIVATIVES FROM
FORCED OSCILLATION WIND TUNNEL
TESTS WITH APPLICATION TO
VOLUTE STABILIZED MODELS**

**AEROMECHANICS BRANCH
TECHNOLOGY DIVISION**

TECHNICAL REPORT AFATL-TR-70-58

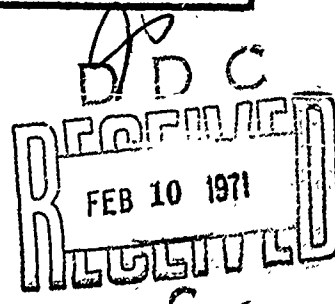
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SOME METHODS FOR EXTRACTING AERODYNAMIC DERIVATIVES
FROM FORCED OSCILLATION WIND TUNNEL TESTS WITH
APPLICATION TO VOLUTE STABILIZED MODELS

Mark O. Glasgow


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FOREWORD

This in-house study was undertaken in support of Project 2547, Task 01, Work Unit 011, "Volute Stabilization of Cylindrical Submunitions." This effort was accomplished during the period February 1970 to June 1970. The author acknowledges the helpful suggestions of Mr. Kenneth K. Cobb and Dr. George B. Findley.

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This technical report has been reviewed and is approved.



CHARLES K. ARPKE, Lt Colonel, USAF
Chief, Technology Division

ABSTRACT

In this report equations are derived for computing static and dynamic aerodynamic derivatives from forced oscillation wind tunnel tests of volume-stabilized munition models. A linear model is used for the pitching moment equation which considers small pitch angles and small volume spring angles (or small perturbations), with no shift in the center of gravity. The theory represents an extension of the methods presented in Arnold Engineering Development Center technical report AEDC-TR-69-208 and in Advisory Group for Aerospace Research and Development report AGARDograph 121 to two degrees of angular freedom.

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LIST OF ABBREVIATIONS AND SYMBOLS

A B C T T ₁ K H	coefficients, constants
C_{m_θ}	$\partial C_m / \partial \theta = C_{m_\alpha}$ pitching moment derivative
C_{m_δ}	$\partial C_m / \partial \delta$ pitch moment derivative
$C_{m_{\dot{\theta}}}$	$C_{m_q} + C_{m_{\dot{\alpha}}}$ pitch damping derivative
$C_{m_{\dot{\delta}}}$	$\partial C_m / \partial \left(\frac{\dot{\delta} d}{2V} \right)$ pitch damping derivative
d	model diameter, reference arm for moments
I	$(I_B + I_t)$ moment of inertia of model about pitch axis
I_B	moment of inertia of forebody about pitch axis
I_t	moment of inertia of volute spring about pitch axis
$I_{t,hinge}$	moment of inertia of volute spring about hinge pitch axis
$\vec{i} \vec{j} \vec{k}$	unit vectors aligned with body axes of forebody
$\tilde{i} \tilde{j} \tilde{k}$	unit vectors aligned with body axes of volute spring
K_t	$(I_{t,hinge} + m_t R_t \bar{r}_t)$
m_i	mass of i^{th} particle
m_t	mass of volute spring
M_θ	stability coefficient
M_θ^*	stability coefficient
M_δ	stability coefficient
M_δ^*	stability coefficient

LIST OF ABBREVIATIONS AND SYMBOLS (continued)

P_i	point with coordinates x_i y_i z_i
q	dynamic pressure = $1/2 \rho V^2$
R_i	separation of i^{th} mass from origin of forebody axes
R_t	separation of volute hinge from origin of forebody axes
\bar{r}_t	$-\frac{1}{m_t} \sum_{tail} m_i x_{ti}$ = separation of hinge origin and tail CG, positive
S	reference area
t	time
V	free stream velocity
x y z	body axes of forebody
x_t y_t z_t	body axes of volute tail
ρ	air density
Δ	value of a determinant in Equation 6
θ	pitch angle or pitch angle perturbation
δ	volute spring angle; angle at hinge between longitudinal axes of forebody and volute spring, or perturbation of this angle
ζ	ratio of damping to critical
α	angle of attack
ω	angular frequency
ω_n	undamped natural frequency of system
ω_v	frequency of forced oscillation in vacuum
θ_o	amplitude of pitch oscillation

LIST OF ABBREVIATIONS AND SYMBOLS (concluded)

δ_0 amplitude of volute angle oscillation

$\psi_1 \psi_2 \phi_1$ phase angles

Superscripts

$(\vec{})$ vector

$(\dot{})$ $d()/dt$

Subscripts

w value with wind on

a aerodynamic value

v value under vacuum conditions

B forebody

t volute tail

i property of i^{th} particle

spring
volute structural property of volute

SECTION 1

INTRODUCTION

Methods have been derived for computing model aerodynamic derivatives from forced oscillation wind tunnel tests with one degree of angular freedom (References 1 and 2). These are for small oscillations in pitch angle of a rigid body. In this report these methods are extended to cover small forced oscillations in the pitch angle of a body with an elastic tail, as with a spring volute stabilized model.

The volute (Figure 1) is a light coiled spring device which stores compactly. In action the coiled spring releases to form a cone-shaped extension to a munition forebody. This rearward extension stabilizes the munition and gives a preferred orientation in applications.

The analysis considers linearized equations of motion for small values of the pitch angle θ and volute angle δ , or small perturbations of these angles.

As the volute is a light, flexible spring, subjected in use to a distributed load, it will bend in an elastic curve. For the purpose of analyzing the inertia torques, this curve is idealized to be a straight line. However, the results are applicable to the real volute, so long as the volute angle δ remains small, with $\cos \delta$ approximately one and no important shifts in the center of gravity of the model or the moments of inertia. For the curved volute, the volute angle δ may be defined as the angle between the longitudinal axis of the forebody and the line connecting the center of the volute hinge with the center of the rear face of the volute spring. The angle δ is taken positive when the volute axis lies below the longitudinal axis of the forebody.

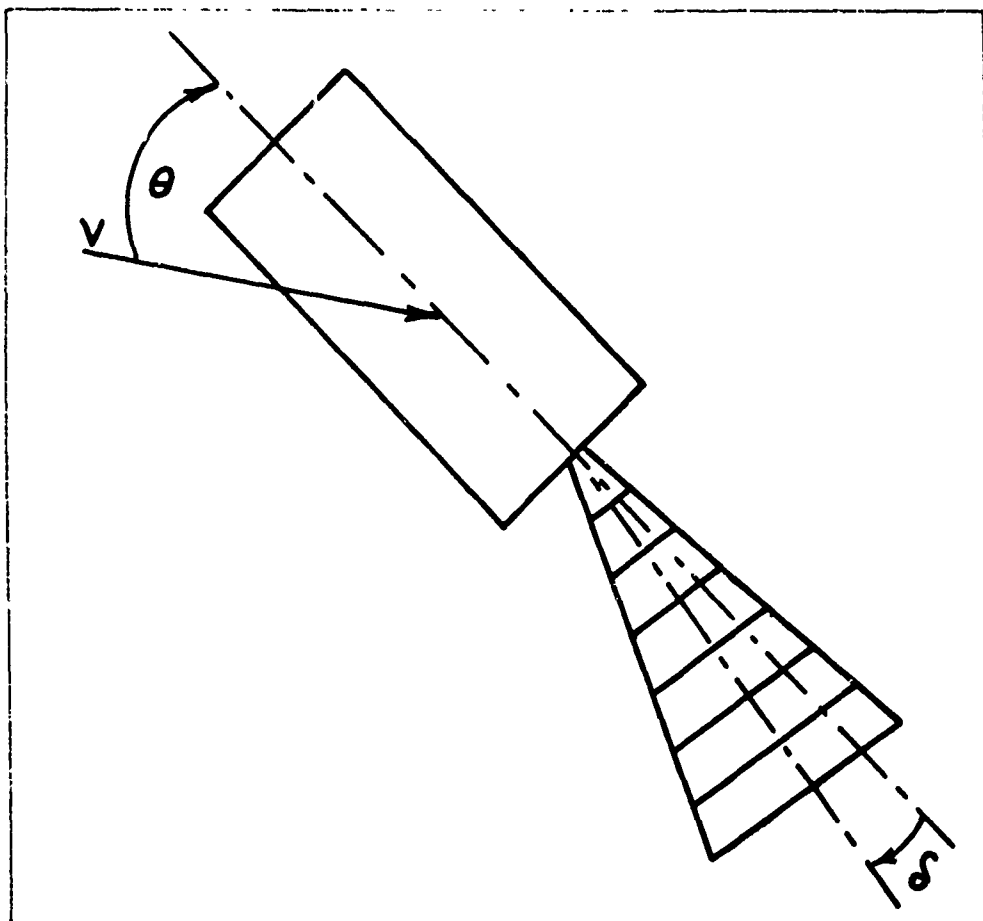


Figure 1. Volute-Stabilized Model with Positive Angles θ and δ .

SECTION II

THE SOLUTION OF AN EQUATION FOR FORCED OSCILLATION OF A LINEAR OSCILLATOR

Consider the equation:

$$\ddot{\theta} + 2\zeta\omega_n\dot{\theta} + \omega_n^2\theta = T\cos\omega t; \quad (0 < \zeta < 1) \quad (1)$$

The solution consists of a solution to the homogeneous equation plus a particular solution. The solution to the homogeneous equation is:

$$\theta_c = [K \exp(-\zeta\omega_n t)] \cos(\omega_n t \sqrt{1-\zeta^2} + \phi) \quad (2)$$

where K and ϕ are functions of initial conditions. The particular solution is:

$$\theta_p = A\sin\omega t + B\cos\omega t \quad (3)$$

Upon substituting $\theta = \theta_c + \theta_p$ into Equation 1 and collecting coefficients, one gets:

$$\begin{aligned} (\cos\omega t) [-\omega^2 B + 2\zeta\omega_n\omega A + \omega_n^2 B] + \sin\omega t \\ [-\omega^2 A - 2\zeta\omega_n\omega B + \omega_n^2 A] = T\cos\omega t \end{aligned} \quad (4)$$

$$-2\zeta\omega_n\omega B + (-\omega^2 + \omega_n^2) A = 0 \quad (5a)$$

$$(-\omega^2 + \omega_n^2) B + 2\zeta\omega_n\omega A = T \quad (5b)$$

$$B = \frac{\begin{vmatrix} 0 & (-\omega^2 + \omega_n^2) \\ T & 2\zeta\omega_n\omega \end{vmatrix}}{\begin{vmatrix} -2\zeta\omega_n\omega & (-\omega^2 + \omega_n^2) \\ (-\omega^2 + \omega_n^2) & 2\zeta\omega_n\omega \end{vmatrix}} = \frac{-T(-\omega^2 + \omega_n^2)}{\Delta} \quad (6a)$$

$$A = -2T\zeta\omega_n\omega/\Delta \quad (6b)$$

If $\omega = \omega_n$, then $B = 0$, $A = T/2\zeta\omega_n^2$, and the steady state value of θ is $\theta = \theta_\rho = \frac{T\sin\omega_n t}{2\zeta\omega_n^2} = \theta_0 \sin\omega_n t$, where θ_0 is the amplitude.

SECTION III

FORCED OSCILLATION AT SYSTEM UNDAMPED NATURAL FREQUENCY (SINGLE DEGREE OF ANGULAR FREEDOM)

This section presents a portion of the theory developed in References 1 and 2 and is included in the interest of completeness. The pitching moment equation may be taken as

$$I\ddot{\theta} - M_{\dot{\theta}}\dot{\theta} - M_{\theta}\theta = T\cos\omega_n t; \quad \omega_n^2 = -M_{\theta}/I \quad (7)$$

where θ is the perturbation of pitch angle, and the perturbation torque is $T\cos\omega_n t$.

After transients have died out

$$\theta = \theta_o \sin\omega_n t \quad (8a)$$

$$\dot{\theta} = \theta_o \omega_n \cos\omega_n t \quad (8b)$$

$$\ddot{\theta} = -\theta_o \omega_n^2 \sin\omega_n t \quad (8c)$$

Substituting into equation 7,

$$-M_{\theta}\theta_o \omega_n^2 = T \quad (9a)$$

$$-M_{\theta}^* = T/\theta_o \omega_n \quad (9b)$$

Since the structural damping coefficient M_{θ}^* is inversely proportional to the frequency

$$M_{\theta_w}^* = M_{\theta_a}^* + M_{\theta_v}^* \left(\frac{\omega_v}{\omega_n} \right); \quad M_{\theta_a}^* = M_{\theta_w}^* - M_{\theta_v}^* \left(\frac{\omega_v}{\omega_n} \right) \quad (10)$$

$$C_{m_{\dot{\theta}}_a} = \frac{2V}{qSd^2} \left[M_{\dot{\theta}_w} - M_{\dot{\theta}_v} \left(\frac{\omega_v}{\omega_n} \right) \right] = C_{m_q} + C_{m_{\dot{\alpha}}} \quad (11)$$

Subscripts:

a = aerodynamic value

v = value under vacuum conditions

w = value with wind on

$$\omega_n = \sqrt{-M_{\theta}/I} \quad (12a)$$

$$M_{\theta_a} = M_{\theta_w} \cdot M_{\theta_v} \quad (12b)$$

$$C_{m_{\theta}} = \frac{1}{qSd} \left[M_{\theta_w} - M_{\theta_v} \right] = C_{m_{\alpha}} \quad (12c)$$

For small amplitudes these are local values of $(C_{m_q} + C_{m_{\dot{\alpha}}})$ and $C_{m_{\alpha}}$. The values of θ (and later δ) are taken as perturbation values, so that Equation 7 may be used to obtain the derivatives over a range of angles of attack.

SECTION IV

INERTIA TERMS IN PITCHING MOMENT EQUATION FOR VOLUTE STABILIZED MODEL WITH TWO DEGREES OF FREEDOM

The model is taken to be rotationally symmetric about the longitudinal axes of the forebody and the tail. Consider a body axis system with origin located on the pitch axis and the longitudinal axis of the forebody, x axis forward along the longitudinal axis, y axis the pitch axis, and x,y,z a right-hand set of axes. Let unit vectors aligned with these axes be $\vec{i}, \vec{j}, \vec{k}$. Let \vec{R}_i be a vector from the origin to the point $P_i(x_i, y_i, z_i)$, $\vec{R}_i = x_i \vec{i} + y_i \vec{j} + z_i \vec{k}$, with m_i the mass at $P_i(x_i, y_i, z_i)$. The angular velocity of the axes is:

$$\vec{\omega} = \dot{\theta} \vec{j}$$

The inertia torque terms are obtained as:

$$\frac{d}{dt} (\overrightarrow{\text{angular momentum}})_B = \frac{d}{dt} \sum_B m_i \vec{R}_i \otimes \dot{\vec{R}}_i \quad (13a)$$

For the forebody:

$$\dot{x}_i = \dot{y}_i = \dot{z}_i = \ddot{x}_i = \ddot{y}_i = \ddot{z}_i = 0$$

$$\dot{\vec{R}}_i = \vec{\omega} \otimes \vec{R}_i = \dot{\theta} [-x_i \vec{k} + z_i \vec{i}]$$

$$(\overrightarrow{\text{angular momentum}})_B = \sum_B m_i \vec{R}_i \otimes \dot{\vec{R}}_i \quad (13b)$$

$$(\overrightarrow{\text{angular momentum}})_B = \dot{\theta} \sum_B m_i [(x_i^2 + z_i^2) \vec{j} - x_i y_i \vec{i} - y_i z_i \vec{k}] \quad (13c)$$

From symmetry,
$$\sum_B m_i (-x_i y_i \vec{i}) = \sum_B m_i (-y_i z_i \vec{k}) = 0$$

$$(\overrightarrow{\text{angular momentum}})_B = \dot{\theta} \vec{j} \sum_B m_i (x_i^2 + z_i^2) \quad (13d)$$

Using the formula, $\frac{d}{dt} (H\vec{j}) = \frac{dH}{dt} \vec{j} + H (\vec{\omega} \otimes \vec{j})$, one notes that

here $\vec{\omega} \otimes \vec{j} = 0$, as there is no rotation about the roll and yaw axes.

Therefore,
$$(\overrightarrow{\text{inertia torque}})_B = \ddot{\theta} \sum_B m_i (x_i^2 + z_i^2) \vec{j} = I_B \ddot{\theta} \vec{j} \quad (13e)$$

where

$$I_B = \sum_B m_i (x_i^2 + z_i^2)$$

To compute the inertia torque of the volute tail about the pitch axis, let an auxiliary set of axes, x_t, y_t, z_t , be located with origin at the tail hinge center (intersection of longitudinal axes of the forebody and tail), with x_t forward along the longitudinal axis of the tail, y_t parallel to the pitch axis, and x_t, y_t, z_t a right-hand set of axes. The x_t axis is inclined to the x axis by the volute angle δ , the volute longitudinal axis is here idealized as a straight line. Let the vector from the origin of x, y, z to the origin of x_t, y_t, z_t be $(-R_t \vec{i})$. Let unit vectors aligned with x_t, y_t, z_t be $\vec{i}, \vec{j}, \vec{k}$. Then for a point $P_i (x_i, y_i, z_i)$ with mass m_i located on the tail, and with (x_t, y_t, z_t) coordinates (x_{ti}, y_{ti}, z_{ti}) .

$$\vec{R}_i = -R_t \vec{i} + x_{ti} \vec{i} + y_{ti} \vec{j} + z_{ti} \vec{k} \quad (14a)$$

$$\vec{R}_i = (\vec{R}_i \cdot \vec{i}) \vec{i} + (\vec{R}_i \cdot \vec{j}) \vec{j} + (\vec{R}_i \cdot \vec{k}) \vec{k} \quad (14b)$$

$$\vec{R}_i = (-R_t + x_{ti} \cos \delta + z_{ti} \sin \delta) \vec{i} + y_{ti} \vec{j} + (-x_{ti} \sin \delta + z_{ti} \cos \delta) \vec{k} \quad (14c)$$

Here

$$\dot{x}_{ti} = \dot{y}_{ti} = \dot{z}_{ti} = \ddot{x}_{ti} = \ddot{y}_{ti} = \ddot{z}_{ti} = 0$$

$$\dot{\vec{R}}_i = \dot{\delta} [(-x_{ti} \sin \delta + z_{ti} \cos \delta) \vec{i} + (-x_{ti} \cos \delta - z_{ti} \sin \delta) \vec{k}] + \vec{\omega} \otimes \vec{R}_i; \quad (14d)$$

$$\vec{\omega} = \vec{j} \dot{\theta}$$

$$\dot{\vec{R}}_1 = \dot{\delta} [(-x_{ti} \sin \delta + z_{ti} \cos \delta) \vec{i} + (-x_{ti} \cos \delta - z_{ti} \sin \delta) \vec{k}] + \quad (14e)$$

$$\dot{\theta} [(-R_t + x_{ti} \cos \delta + z_{ti} \sin \delta) \vec{k} + (-x_{ti} \sin \delta + z_{ti} \cos \delta) \vec{i}]$$

$$(\overrightarrow{\text{angular momentum}})_t = \sum_t m_i \vec{R}_i \otimes \dot{\vec{R}}_i \quad (14f)$$

From symmetry, the terms in the summation involving y_{ti} drop out.

$$\begin{aligned} (\overrightarrow{\text{angular momentum}})_t = \sum_t m_i \left\{ \dot{\delta} [(-R_t + x_{ti} \cos \delta + z_{ti} \sin \delta) \cdot \right. \\ (-x_{ti} \cos \delta - z_{ti} \sin \delta) (-\vec{j}) + (-x_{ti} \sin \delta + z_{ti} \cos \delta) (-x_{ti} \sin \delta + z_{ti} \cos \delta) \vec{j}] \\ + \dot{\theta} [(-R_t + x_{ti} \cos \delta + z_{ti} \sin \delta) (-R_t + x_{ti} \cos \delta + z_{ti} \sin \delta) \\ \left. + (-x_{ti} \sin \delta + z_{ti} \cos \delta) (-x_{ti} \sin \delta + z_{ti} \cos \delta) \vec{j}] \right\} \quad (14g) \end{aligned}$$

$$\begin{aligned}
(\overrightarrow{\text{angular momentum}})_t &= \vec{j} \sum_t m_i \left\{ \dot{\delta} [-R_t x_{ti} \cos \delta - R_t z_{ti} \sin \delta \right. \\
&\quad \left. + x_{ti}^2 + z_{ti}^2] + \dot{\theta} [R_t^2 - 2R_t x_{ti} \cos \delta + x_{ti}^2 + z_{ti}^2 - 2R_t z_{ti} \sin \delta] \right\}
\end{aligned} \tag{14h}$$

We note that $-\sum_t m_i \dot{\delta} R_t z_{ti} \sin \delta = -2 \sum_t m_i \dot{\theta} R_t z_{ti} \sin \delta = 0$ because of symmetry.

$$\begin{aligned}
(\overrightarrow{\text{angular momentum}})_t &= \vec{j} \sum_t m_i \left\{ \dot{\delta} [-R_t x_{ti} \cos \delta + x_{ti}^2 + z_{ti}^2] \right. \\
&\quad \left. + \dot{\theta} [R_t^2 - 2R_t x_{ti} \cos \delta + x_{ti}^2 + z_{ti}^2] \right\}
\end{aligned} \tag{14i}$$

$$(\overrightarrow{\text{inertia torque}})_t = \frac{d}{dt} (\overrightarrow{\text{angular momentum}})_t \tag{15a}$$

$$\begin{aligned}
(\overrightarrow{\text{inertia torque}})_t &= \vec{j} \sum_t m_i \left\{ \ddot{\delta} [-R_t x_{ti} \cos \delta + x_{ti}^2 + z_{ti}^2] \right. \\
&\quad \left. + \ddot{\theta} [R_t^2 - 2R_t x_{ti} \cos \delta + x_{ti}^2 + z_{ti}^2] + \dot{\delta}^2 [R_t x_{ti} \sin \delta] + \dot{\theta} \dot{\delta} [2R_t x_{ti} \sin \delta] \right\}
\end{aligned} \tag{15b}$$

Assuming δ , $\dot{\theta}$, and $\dot{\delta}$ remain small, the last two brackets are higher order terms and may be neglected.

$$\begin{aligned}
(\overrightarrow{\text{inertia torque}})_t &= \vec{j} \sum_t m_i \left\{ \ddot{\delta} [-R_t x_{ti} \cos \delta + x_{ti}^2 + z_{ti}^2] \right. \\
&\quad \left. + \ddot{\theta} [R_t^2 - 2R_t x_{ti} \cos \delta + x_{ti}^2 + z_{ti}^2] \right\}
\end{aligned} \tag{15c}$$

Here

$$-\sum_t m_i R_t x_{ti} \cos \delta = m_t R_t \bar{r}_t \cos \delta$$

$$\sum_t m_i (x_{ti}^2 + z_{ti}^2) = I_{t,hinge}$$

$$-\sum_t m_i (2R_t x_{ti} \cos \delta) = 2m_t R_t \bar{r}_t \cos \delta$$

where $\sum_t m_i = m_t$ and \bar{r}_t is the separation of the tail CG from the tail origin at the hinge.

Abbreviating

$$\sum_t m_i [-R_t x_{ti} \cos \delta + x_{ti}^2 + z_{ti}^2] = m_t R_t \bar{r}_t \cos \delta + I_{t,hinge} = K_t$$

$$\sum_t m_i [R_t^2 - 2R_t x_{ti} \cos \delta + x_{ti}^2 + z_{ti}^2] = m_t R_t^2 + 2m_t R_t \bar{r}_t \cos \delta + I_{t,hinge} = I_t$$

$$I = I_B + I_t$$

The inertia torque of the model is then given by

$$\text{Inertia torque} = I \ddot{\theta} + K_t \ddot{\delta} \quad (16)$$

SECTION V

WIND TUNNEL FORCED OSCILLATION AT UNDAMPED NATURAL FREQUENCY OF SYSTEM WITH TWO DEGREES OF ANGULAR FREEDOM, LINEAR MODEL

The linearized pitching moment equation ^(1,2) may be taken as

$$I\ddot{\theta} + K_t\ddot{\delta} - M_{\dot{\theta}}\dot{\theta} - M_{\dot{\delta}}\dot{\delta} - M_{\theta}\theta - M_{\delta}\delta = T\cos\omega t \quad (17)$$

where θ and δ are perturbations of the pitch and volute angles, and the perturbing moment is $T\cos\omega t$. Here the unperturbed value of the volute angle δ is used in evaluating the constant values of I and K_t . For the wind-on condition in the tunnel, the coefficients would have a subscript w and for vacuum conditions, these coefficients would have a subscript v .

In the dynamic tests, the model is subjected to a forcing torque, $T\cos\omega t$. Assuming that $\dot{\theta}$ and $\dot{\delta}$ are positively damped ($M_{\dot{\theta}} < 0$, $M_{\dot{\delta}} < 0$), the transient solution to the homogeneous equation damps out and disappears from the motion. The remaining motions of θ and δ are then sinusoidal vibrations at frequency ω . Equation 17 may then be split into separate equations for θ and δ as follows:

$$I\ddot{\theta} - M_{\dot{\theta}}\dot{\theta} - M_{\theta}\theta = A\cos\omega t + B\sin\omega t \quad (18)$$

$$K_t\ddot{\delta} - M_{\dot{\delta}}\dot{\delta} - M_{\delta}\delta = (T-A)\cos\omega t - B\sin\omega t \quad (19)$$

Here A and B remain to be determined.

The technique of forcing at the undamped natural frequency ω_n of the system, where $\omega_n^2 = -M_{\theta}/I$, is to vary ω until $\theta = \theta_0 \sin\omega t$ ($= \theta_0 \sin\omega_n t$); ω_n occurs when the phase angle between θ and the forcing moment is 90° . This drives B to a value of zero.

For $\omega = \omega_n$

$\theta = \theta_0 \sin \omega_n t$ (here θ_0 is the amplitude)

$$\dot{\theta} = \theta_0 \omega_n \cos \omega_n t \quad (20)$$

$$\ddot{\theta} = -\theta_0 \omega_n^2 \sin \omega_n t$$

$$I\ddot{\theta} - M_{\theta}^* \dot{\theta} - M_{\theta} \theta = -M_{\theta}^* \theta_0 \omega_n \cos \omega_n t = A \cos \omega_n t$$

$$M_{\theta}^* = -A / \theta_0 \omega_n \quad (21)$$

Using the subscripts

a = aerodynamic value

w = value with wind on

v = value under vacuum conditions

one gets

$$M_{\theta_w}^* = -A / \theta_0 \omega_n \quad (22)$$

$$M_{\theta_a}^* = M_{\theta_w}^* - M_{\theta_v}^* \left(\frac{\omega_v}{\omega_n} \right) \quad (23)$$

$$\left(C_{m_q} + C_{m_{\dot{\alpha}}} \right) = M_{\theta_a}^* \left(\frac{2V}{qSd^2} \right) \quad (24)$$

$$M_{\theta_a} = M_{\theta_w} - M_{\theta_v}; \quad M_{\theta_w} = -I\omega_n^2 \quad (25)$$

$$C_{m_{\alpha}} = M_{\theta_a} / qSd \quad (26)$$

The wind on values of the coefficients will be known when the quantity A has been evaluated and the vacuum values have been determined.

At the same time, $\delta = \delta_o \cos(\omega_n t + \phi)$ where δ_o is the amplitude, ϕ is the phase angle, and δ_o and ϕ may be measured.

$$\delta = \delta_o \cos(\omega_n t + \phi)$$

$$\dot{\delta} = -\omega_n \delta_o \sin(\omega_n t + \phi) \quad (27)$$

$$\ddot{\delta} = -\omega_n^2 \delta_o \cos(\omega_n t + \phi)$$

$$K_t \ddot{\delta} - M_\delta \dot{\delta} - M_\delta \delta = (T-A) \cos \omega_n t \quad (28)$$

$$\begin{aligned} -K_t \omega_n^2 \delta_o \cos(\omega_n t + \phi) + M_\delta \omega_n \delta_o \sin(\omega_n t + \phi) \\ -M_\delta \delta_o \cos(\omega_n t + \phi) = (T-A) \cos \omega_n t \end{aligned} \quad (29)$$

$$\begin{aligned} -K_t \omega_n^2 \delta_o (\cos \phi \cos \omega_n t - \sin \phi \sin \omega_n t) + M_\delta \omega_n \delta_o (\sin \phi \cos \omega_n t \\ + \cos \phi \sin \omega_n t) - M_\delta \delta_o (\cos \phi \cos \omega_n t - \sin \phi \sin \omega_n t) = (T-A) \cos \omega_n t \end{aligned} \quad (30)$$

Upon equating the coefficients of $\cos \omega_n t$ and $\sin \omega_n t$ on the two sides of the equation, there results

$$\begin{aligned} K_t \omega_n^2 \delta_o \sin \phi + M_\delta \omega_n \delta_o \cos \phi + M_\delta \delta_o \sin \phi = 0 \\ -K_t \omega_n^2 \delta_o \cos \phi + M_\delta \omega_n \delta_o \sin \phi - M_\delta \delta_o \cos \phi = T-A \end{aligned} \quad (31)$$

On rearranging,

$$M_\delta (\omega_n \cos \phi) + M_\delta (\sin \phi) = -K_t \omega_n^2 \sin \phi \quad (32)$$

$$M_{\delta}^*(\omega_n \delta_o \sin\phi) - M_{\delta}(\delta_o \cos\phi) = (T-A) + K_t \omega_n^2 \delta_o \cos\phi \quad (33)$$

whence

$$M_{\delta}^* = \frac{\begin{vmatrix} -K_t \omega_n^2 \sin\phi & \sin\phi \\ (T-A) + K_t \omega_n^2 \delta_o \cos\phi & -\delta_o \cos\phi \end{vmatrix}}{-\omega_n \delta_o} \quad (34)$$

$$M_{\delta} = \frac{\begin{vmatrix} \omega_n \cos\phi & -K_t \omega_n^2 \sin\phi \\ \omega_n \delta_o \sin\phi & (T-A) + K_t \omega_n^2 \delta_o \cos\phi \end{vmatrix}}{-\omega_n \delta_o} \quad (35)$$

On expanding the determinants,

$$M_{\delta}^* = \frac{(T-A)\sin\phi}{\omega_n \delta_o} \quad (36)$$

$$M_{\delta} = \frac{-[(T-A)\cos\phi + K_t \omega_n^2 \delta_o]}{\delta_o} \quad (37)$$

Solving for (T-A),

$$T-A = \frac{\omega_n \delta_o M_{\delta}^*}{\sin\phi} = \frac{-\delta_o M_{\delta} - K_t \omega_n^2 \delta_o}{\cos\phi} \quad (38)$$

In order to evaluate (T-A), one method would be a static determination of M_δ so that the far right member of equation 38 could be evaluated. Another method would be forced oscillation at the undamped natural frequency of the volute. One has

$$M_{\delta_w} = M_{\delta_a} + M_{\delta_{spring}} = \frac{\text{Static Moment}}{\delta} \quad (39)$$

whence

$$C_{m_\delta} = M_{\delta_a} / qSd \quad (40)$$

$$A = T - (T-A) \quad (41)$$

$$M_{\delta_w}^* = \frac{(T-A)\sin\phi}{\omega_n \delta_o} = M_{\delta_a}^* + M_{\delta_{spring}}^* \quad (42a)$$

$$M_{\delta_{spring}}^* = M_{\delta_v}^* \left(\frac{\omega_v}{\omega_n} \right) \quad (42b)$$

$$C_{m_\delta}^* = \left(M_{\delta_w}^* - M_{\delta_{spring}}^* \right) \frac{2V}{qSd^2} = M_{\delta_a}^* \left(\frac{2V}{qSd^2} \right) \quad (43)$$

There remains the problem of obtaining $M_{\delta_v}^*$ and M_{θ_v} . The model is oscillated in the vacuum tunnel at the undamped natural frequency of the system. The equation is

$$I\ddot{\theta} + K_t\ddot{\delta} - M_{\theta}^*\ddot{\theta} - M_{\delta}^*\ddot{\delta} - M_{\theta\delta}^*\ddot{\theta} - M_{\delta\theta}^*\ddot{\delta} = T_1 \cos\omega t \quad (44)$$

(similar to Equation 17)

After the transients have died out and $\omega = \omega_{n_V}$, one has

$$(I\ddot{\theta} - M_{\dot{\theta}}\dot{\theta} - M_{\theta}\theta)_V = C \cos \omega_{n_V} t \quad (45)$$

$$K_t \ddot{\delta} - M_{\dot{\delta}}\dot{\delta} - M_{\delta}\delta = (T_1 - C) \cos \omega_{n_V} t \quad (46)$$

$$\theta = \theta_0 \sin \omega_{n_V} t$$

$$\dot{\theta} = \omega_{n_V} \theta_0 \cos \omega_{n_V} t \quad (47)$$

$$\ddot{\theta} = -\omega_{n_V}^2 \theta_0 \sin \omega_{n_V} t$$

$$(-M_{\dot{\theta}})_V \omega_{n_V} \theta_0 = C \quad (48a)$$

$$M_{\dot{\theta}} = -C / \omega_{n_V} \theta_0 \quad (48b)$$

$$M_{\theta} = -I \omega_{n_V}^2 \quad (49)$$

At the same time, $\delta = \delta_0 \cos(\omega_{n_V} t + \phi_1)$ (by analogy with Equation 36)

$$M_{\dot{\delta}} = \frac{(T_1 - C) \sin \phi_1}{\omega_{n_V} \delta_0} \quad (50)$$

$$M_{\delta} = \frac{-[(T_1 - C) \cos \phi_1 + K_t \omega_{n_V}^2 \delta_0]}{\delta_0} \quad (51)$$

$$T_1 - C = \frac{-\delta_o M_{\delta_v} - K_t \omega_{n_v}^2 \delta_o; C = T_1 - (T_1 - C)}{\cos \phi_1} \quad (52)$$

The restoring moment coefficient of the spring volute M_{δ_v} , may be obtained by static experiment, or from forced oscillation at the undamped natural frequency of the volute, so that C may be obtained from Equation 52, and M_θ and M_{θ_v} from Equations 48 and 49.

SECTION VI

WIND TUNNEL FORCED OSCILLATION AT A FREQUENCY OTHER THAN ω_n ; TWO DEGREES OF ANGULAR FREEDOM, LINEAR MODEL

The equation of motion may be taken as (same as Equation 17)

$$I\ddot{\theta} + K_t\ddot{\delta} - M_\theta\dot{\theta} - M_\delta\dot{\delta} - M_\theta\theta - M_\delta\delta = T\cos\omega t \quad (53)$$

After decay of the transients, there results

$$I\ddot{\theta} - M_\theta\dot{\theta} - M_\theta\theta = A\cos\omega t + B\sin\omega t \quad (54)$$

$$K_t\ddot{\delta} - M_\delta\dot{\delta} - M_\delta\delta = (T-A)\cos\omega t - B\sin\omega t \quad (55)$$

where A&B remain to be evaluated.

The remaining motions are

$$\theta = \theta_0 \cos(\omega t + \psi_1) \quad (56)$$

$$\delta = \delta_0 \cos(\omega t + \psi_2) \quad (57)$$

where ψ_1, ψ_2, θ_0 , and δ_0 may be evaluated from the test. Substituting from Equation 56 into Equation 54

$$-I\theta_0\omega^2 [\cos\psi_1 \cos\omega t - \sin\psi_1 \sin\omega t] + M_\theta\theta_0\omega [\sin\psi_1 \cos\omega t + \cos\psi_1 \sin\omega t] \quad (58)$$

$$-M_\theta\theta_0 [\cos\psi_1 \cos\omega t - \sin\psi_1 \sin\omega t] = A\cos\omega t + B\sin\omega t$$

Upon equating the coefficients of $\sin\omega t$ and $\cos\omega t$ on the two sides of Equation 58, there results

$$M_\theta(\omega\theta_0 \cos\psi_1) + M_\theta\theta_0 \sin\psi_1 = -I\omega^2\theta_0 \sin\psi_1 + B \quad (59)$$

$$M_\theta(\omega\theta_0 \sin\psi_1) - M_\theta(\theta_0 \cos\psi_1) = A + I\omega^2\theta_0 \cos\psi_1 \quad (60)$$

$$M_{\theta}^* = \frac{\begin{vmatrix} -I\omega^2 \theta_o \sin\psi_1 + B & \theta_o \sin\psi_1 \\ A + I\omega^2 \theta_o \cos\psi_1 & -\theta_o \cos\psi_1 \end{vmatrix}}{-\omega \theta_o^2} \quad (61)$$

$$M_{\theta} = \frac{\begin{vmatrix} \omega \theta_o \cos\psi_1 & -I\omega^2 \theta_o \sin\psi_1 + B \\ \omega \theta_o \sin\psi_1 & A + I\omega^2 \theta_o \cos\psi_1 \end{vmatrix}}{-\omega \theta_o^2} \quad (62)$$

On expanding these determinants and simplifying there results

$$M_{\theta}^* = \frac{A \sin\psi_1 + B \cos\psi_1}{\omega \theta_o} \quad (63)$$

$$M_{\theta} = \frac{-[A \cos\psi_1 + I\omega^2 \theta_o - B \sin\psi_1]}{\theta_o} \quad (64)$$

Upon substituting from Equation 57 into Equation 55, there results

$$\begin{aligned} & -K_t \delta_o \omega^2 [\cos\psi_2 \cos\omega t - \sin\psi_2 \sin\omega t] + M_{\theta}^* \delta_o \omega [\sin\psi_2 \cos\omega t + \cos\psi_2 \sin\omega t] \\ & - M_{\theta} \delta_o [\cos\psi_2 \cos\omega t - \sin\psi_2 \sin\omega t] = (T-A) \cos\omega t - B \sin\omega t \end{aligned} \quad (65)$$

Upon equating the coefficients of $\sin\omega t$ and $\cos\omega t$ on the two sides of this equation and rearranging, one gets

$$M_{\delta}(\omega\delta_o \cos\psi_2) + M_{\delta} \delta_o \sin\psi_2 = -K_t \omega^2 \delta_o \sin\psi_2 - B \quad (66)$$

$$M_{\delta}(\omega\delta_o \sin\psi_2) - M_{\delta} \delta_o \cos\psi_2 = (T-A) + K_t \omega^2 \delta_o \cos\psi_2 \quad (67)$$

Solving,

$$M_{\delta}^* = \frac{\begin{vmatrix} -K_t \omega^2 \delta_o \sin\psi_2 - B & \delta_o \sin\psi_2 \\ (T-A) + K_t \omega^2 \delta_o \cos\psi_2 & -\delta_o \cos\psi_2 \end{vmatrix}}{-\omega\delta_o^2} \quad (68)$$

$$M_{\delta} = \frac{\begin{vmatrix} \omega\delta_o \cos\psi_2 & -K_t \omega^2 \delta_o \sin\psi_2 - B \\ \omega\delta_o \sin\psi_2 & (T-A) + K_t \omega^2 \delta_o \cos\psi_2 \end{vmatrix}}{-\omega\delta_o^2} \quad (69)$$

Simplifying,

$$M_{\delta}^* = \frac{(T-A)\sin\psi_2 - B\cos\psi_2}{\omega\delta_o} \quad (70)$$

$$M_{\delta} = \frac{-[(T-A)\cos\psi_2 + K_t \omega^2 \delta_o + B\sin\psi_2]}{\delta_o} \quad (71)$$

Further information is needed to evaluate A and B. If $M_{\delta_w} = M_{\delta_a} + M_{\delta_{spring\ volute}}$ is known from static tests, Equation 71 is one equation involving A and B. Another relation is Equation 64, if $M_{\theta_w} = M_{\theta_a} + M_{\theta_v}$, and M_{θ_a} is known from static tests, and M_{θ_v} is known from vacuum tests. Assuming these values are known,

$$A \cos \psi_1 - B \sin \psi_1 = -M_{\theta_o} - I \omega^2 \theta_o \quad (72)$$

$$-A \cos \psi_2 + B \sin \psi_2 = -M_{\delta_o} - T \cos \psi_2 - K_t \omega^2 \delta_o \quad (73)$$

$$A = \frac{\begin{vmatrix} -M_{\theta_o} - I \omega^2 \theta_o & -\sin \psi_1 \\ -M_{\delta_o} - T \cos \psi_2 - K_t \omega^2 \delta_o & \sin \psi_2 \end{vmatrix}}{\sin(\psi_2 - \psi_1)} \quad (74)$$

$$B = \frac{\begin{vmatrix} \cos \psi_1 & -M_{\theta_o} - I \omega^2 \theta_o \\ -\cos \psi_2 & -M_{\delta_o} - T \cos \psi_2 - K_t \omega^2 \delta_o \end{vmatrix}}{\sin(\psi_2 - \psi_1)} \quad (75)$$

It may be observed that when $\omega = \omega_n$, $\psi_1 = -90^\circ$, $B = 0$, and $(T-A)$ has the same value as that given by Equation 38.

These values of A and B may be substituted into Equations 61 and 62 to evaluate $M_{\theta_w}^*$ and M_{δ_w} . The quantities $M_{\theta_v}^*$, M_{θ_v} , and M_{δ_v} may be obtained as in Section V by vacuum tests at the system undamped natural frequency.

They may also be substituted into Equations 70 and 71 to get $M_{\delta_w}^*$ and $M_{\delta_v}^*$; however, M_{θ_w} and M_{δ_w} have been assumed known in this application from static tests and vacuum tests.

The remaining equations for the aerodynamic derivatives are the following:

$$M_{\theta_a}^* = M_{\theta_w}^* - M_{\theta_v}^* \left(\frac{\omega_v}{\omega} \right) \quad (76)$$

$$\left(C_{m_q} + C_{m_{\alpha}}^* \right) = M_{\theta_a}^* \left(\frac{2V}{qSd^2} \right) \quad (24)$$

$$M_{\theta_a} = M_{\theta_w} - M_{\theta_v} \quad (25)$$

$$C_{m_{\alpha}} = M_{\theta_a} / qSd \quad (26)$$

$$M_{\delta_a}^* = M_{\delta_w}^* - M_{\delta_v}^* \text{ spring; } M_{\delta_{\text{spring}}}^* = M_{\delta_v}^* \left(\frac{\omega_v}{\omega} \right) \quad (77)$$

volute volute

$$C_{m_{\delta}}^* = M_{\delta_a}^* (2V/qSd^2) \quad (78)$$

$$M_{\delta_a} = M_{\delta_w} - M_{\delta_{\text{spring}}} \text{ volute} \quad (79)$$

$$C_{m_{\delta}} = M_{\delta_a} / qSd \quad (40)$$

The restrictions on the above solutions are

1. Configurations with symmetry about the pitch and yaw planes or greater symmetry.
2. First order linear aerodynamics.
3. Small perturbations.
4. No significant changes in CG location or moments of inertia during the motion.
5. System damping causes initial condition transients to decay.

SECTION VII

CONCLUSION

Equations have been obtained to compute volute-equipped munition model aerodynamic derivatives from forced oscillation wind tunnel tests of pitching motions. These methods represent extensions of the methods outlined in References 1 and 2. They require certain additional values of coefficients which may be obtained from static and vacuum tests in the wind tunnel. Structural and aerodynamic contributions to pitch damping and overturning moment coefficients may be computed separately by the process outlined.

It is believed that within the limitations of the linear theory outlined, these equations should serve in reducing data from forced oscillation wind tunnel tests of volute-stabilized models of munitions.

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